Provisional Investigation of Geometric Proportions in 15th-century Cologne Panel Painting

Robert Bork

Geometrical analyses of art have a mixed reputation. In very general terms, it is clear that artists and craftsmen throughout the centuries have had to use geometrical techniques of one kind or another to perform even basic tasks like setting out a building site, arranging the layout of a book page, or cutting a wooden panel into rectangular shape for a painting project. But, of course, this does not imply that such techniques also went on to govern the way painters and other artists established the details of their compositions. For this reason, attempts to explain the proportions and arrangements of elements within paintings have often been met with justifiable skepticism.

Speaking personally, I can recall the mix of fascination and suspicion I felt as a child when I first encountered books like Matila Ghyka's 1946 study "The Geometry of Art and Life," which purported to explain the forms of everything from seashells and human faces to paintings and cathedrals in terms of the same basic principles.[1] I could clearly see that the spiral shell of a chambered nautilus grew in accord with a simple geometric principle related to so-called Golden Section, but I could not readily accept the assertion that the Golden Section and its derivatives underlay the wide range of natural and man-made forms that its champions claimed.[2] Later, as a student, I saw for the first time a series of complex and seemingly overwrought geometrical analyses of paintings, some of which I now recognize as illustrations from Charles Bouleau's 1963 book "The Painter's Secret Geometry."[3] Again, my reaction was skeptical, and I recall thinking that the creation of such diagrams involved a great deal of wishful thinking, since it seemed that almost any element in a painting could be shown to be "significant" if enough lines were drawn over the painting. Since many artists are clearly capable of representing the real world quite effectively without recourse to such proportioning grids, I found myself taking a rather dismissive attitude to such research. I never expected, therefore, to write an essay like this one, which argues that simple geometrical constructions actually do underlie the proportions of fifteenth-century panel paintings from Cologne, governing not only the shape of the panels themselves, but also their figural compositions and their relationship to their architectural frames.

To put this argument into context, I should explain briefly how I came to the study of medieval geometry. My interest in medieval art, from the beginning, has always centered on Gothic architecture. Buildings like Cologne Cathedral impress me greatly, not only because of their large scale, but also because of their formal complexity, which clearly involves intricate geometrical relationships. This can be seen most obviously in the compass-drawn patterns of window tracery, but is is also plainly evident in the elaborately faceted plans of pinnacles, spires, and even whole buildings. When I began to explore the literature on Gothic architecture, though, I found that there was little scholarly agreement about the geometrical methods used by Gothic designers. More surprisingly still, little sustained attention has been paid to the topic since 1970. This neglect reflects the impact of Konrad Hecht's "Mass und Zahl in der gotischen Baukunst," a densely argued polemical study suggesting that arithmetical rather than geometrical thinking was fundamental to Gothic design.[4] Over the past decade, I have undertaken a broadly-ranging investigation of Gothic architectural drawings, which has convinced me that Hecht was mistaken. In fact, his proposals really amount to little more than quantified description. While Gothic builders certainly used repeating modular systems in some contexts, the proportions and detailing of Gothic Gothic drawings and buildings can only be adequately explained by the progressive unfolding of geometry. I therefore see my new book "The Geometry of Creation" as a rejoinder to Hecht, and I hope that its publication will help to stimulate a revived scholarly dialog about Gothic design practice.[5]

Consideration of the great drawing known as Cologne Plan F helps to explain this approach to design, thus setting the stage for consideration of the panel paintings produced in Cologne later in the Middle Ages. Plan F depicts the façade of Cologne Cathedral, complete from left to right and from its base to the top of its twin openwork spires. The architecture shown in Plan F is perfectly symmetrical, but the arrangment

of parchment pieces that comprise the drawing is not, as Figure 1 shows. The left-hand tower is drawn on a stack of five main parchments that tapers slowly, but the right-hand tower is drawn on a stack of six parchments that tapers rather suddenly, like a wine bottle. The right tower was probably drawn first, since the latter arrangement is more geometrically fundamental, in that each of the six pieces aligns with a story of the façade depicted in the drawing. The width of the upper right parchment, moreover, relates closely to octagonal geometry of the spire base. This form, in turn, relates via a series of nested circular constructions to a set of larger octagons, each one of which corresponds to the elevation of a major tower story. [6] The lowest story of the facade appears shorter than the others, because the lowest section of its framing octagon lies underground, indicating that it was circumscribed about a generating square rising from the ground line and framed by the main tower buttresses, as the bottom right portion of Figure 1 shows.

In the context of this essay, these details are less important than several general points that emerge from the analysis: first, that Gothic designers used dynamic geometry to determine the proportions and articulation of their buildings; second, that regular polygons and circular arcs were prominent elements in their geometrical toolkit; and third, that there could be surprisingly intimate relationships between the architectural designs and the format of the parchment pieces on which they were drawn. It is worth emphasizing, moreover, that this analysis depends principally on consideration of lines plainly visible on the parchment. In some instances Gothic draftsmen left blind lines and compass prick marks on their drawings, but these are far more common on ground plans than they are on elevation drawings, which were evidently meant to have more visual appeal. It is obvious that Gothic draftsmen used geometrical tools, but it also clear that they could minimize the traces of their work when they wanted to.[7] It therefore makes good sense to explore the geometry of the lines that they actually inked, rather than giving up on the analysis simply because few blind lines are visible. In all of these respects, the lessons learned in the study of Gothic architectural drawings can be fruitfully applied to the study of panel paintings.

In the summer of 2010, I was invited to investigate the geometry of two paintings in the collection of the Wallraf-Richartz Museum: a large Crucifixion attributed to the circle of the Veronica Master; and a smaller depiction of Saints Mark, Barbara, and Luke, attributed to Stefan Lochner (Figures 2 and 4).[8] Because of my background in architecture, I decided to start by examining the gabled tracery rows that serve as the top frames for each panel. To my surprise, I found that the geometry established in these architectural zones could readily be extended to give valuable information about the proportions of the panels, and about the arrangement of the figures depicted on the panels. Although I was working independently, I later realized that these relationships between figure and frame recall those seen in the work of Bouleau, which I now see as having far more value than I had supposed as a student.[9] In the next portion of of this essay, I will present the results of my admittedly very preliminary investigation, hoping once again to contribute to a growing scholarly dialog on the geometry of medieval art. In the graphics that follow, my first steps will be shown in red, while later steps will follow the spectral pattern of the rainbow, moving through orange, yellow, green, blue, and violet. This color-coding will hopefully make the graphics comprehensible even in the final stages where many colored lines will cross each other.

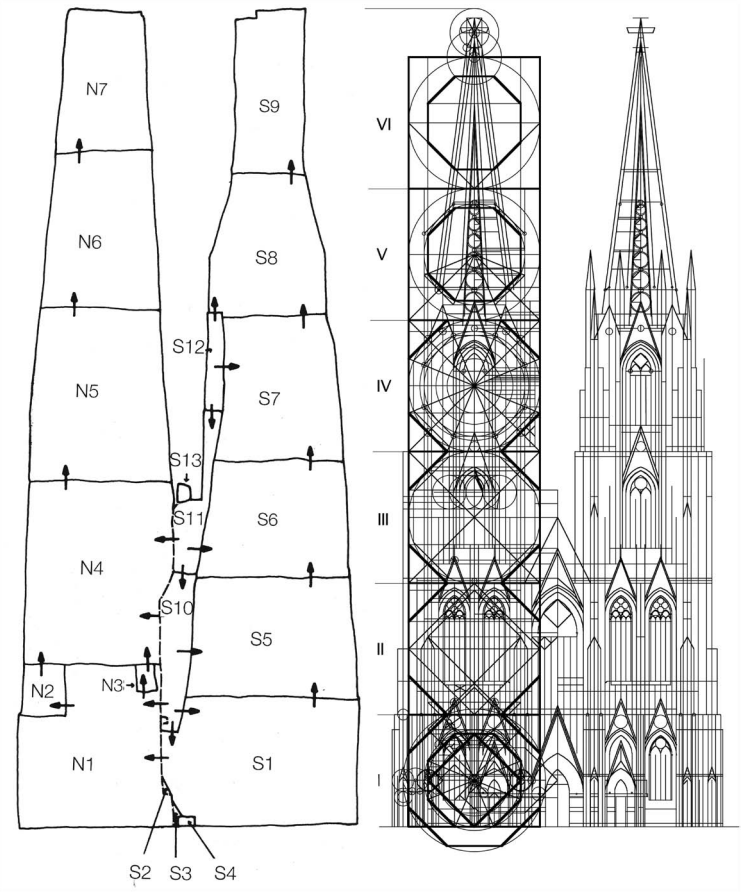


Figure 1 Schematic view of parchment pieces in Cologne Plan F (at left), and geometrical organization of Cologne Cathedral façade depicted in the drawing (at right), showing correlation between parchment structure and stacked octagonal modules governing the façade elevation.

The crucifixion panel, which surely formed the center of an altarpiece, shows a row of saints standing against a golden background, with the cross evenly subdividing the composition.[10] To the left of Christ stand Saints James the Lesser, Andrew, Peter, and the Virgin embraced by John the Evangelist; to the right stand Paul, Bartholomew, Thomas, and Philip. Two small angels fly underneath Christ's outstretched arms, while four others fly a bit higher, just beneath the traceried frieze.

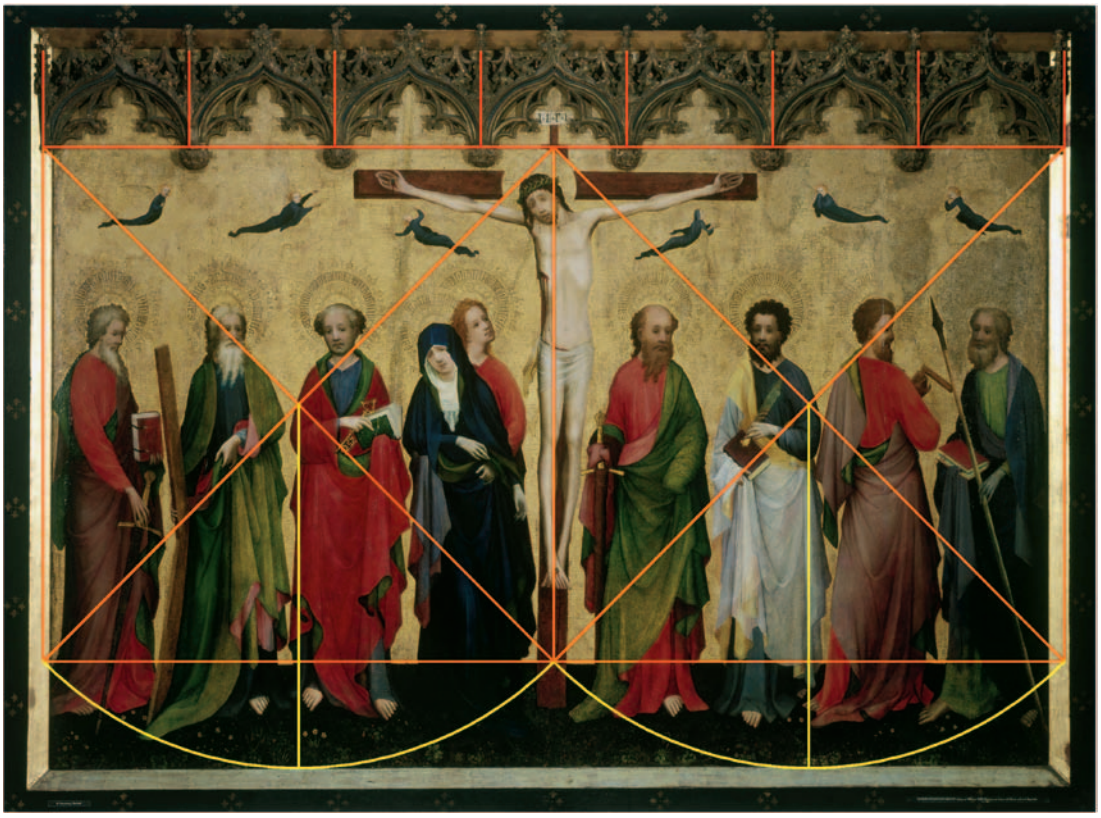


Figure 2 Veronica Master, *Crucifixion*, with geometrical overlay, stage 1.

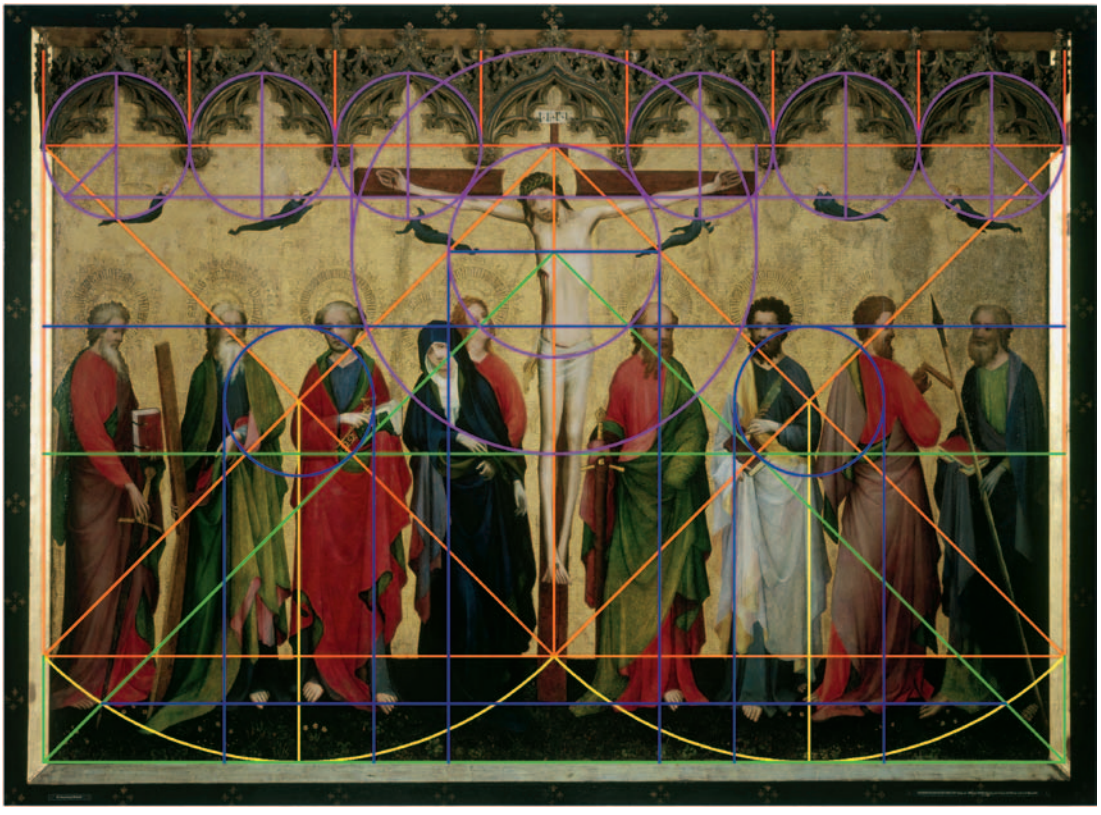


Figure 3 Veronica Master, *Crucifixion*, with geometrical overlay, stage 2.

Figure 4 Stefan Lochner, *Saints Mark, Barbara, and Luke*, with geometrical overlay, stage 1.

Figure 5 Stefan Lochner, *Saints Mark, Barbara, and Luke*, with geometrical overlay, stage 2.

Figure 6 Stefan Lochner, Saints Mark, Barbara, and Luke, with geometrical overlay, stage 3.

To analyse this scene, I began by establishing the baseline from which all the arches in the frieze spring. The next obvious step was to locate the vertical axes of the pinnacles between the traceried arches, because such axes are fundamental to Gothic architectural design. As the red lines in the top portion of Figure 2 indicate, the outer pinnacles axes stand measurably beyond the boundaries of the painted panel in its present frame. But, as the lower portions of the figure show, the dimensions expressed in the traceried arches relate very closely indeed to the organization of the painting itself. If we strike orange diagaonals down from the outermost points on the tracery, we find that they converge at the foot of the cross. The gold background between the arches and the groundline is thus a perfect double square, measuring once again between the outer pinnacle axes. The bottom margin of the panel can be found by circumscribing circles around these squares; the yellow arcs through the corners of the squares describe the bottom quarters of such circles. The radii of the circles, in other words, correspond to the yellow verticals here dropping from the square centers. The left one aligns with the left margin of Saint Peter's red cloak, while the right one similarly aligns with the right margin of Saint Bartholomew's white cloak. These alignments might at first seem coincidental, or simple expressions of the fact that these verticals divide the panel into quarters, but the relationship between the figures and the geometry of the panel is too precise to be dismissed.

Further confirmation for the relevance of the panel dimensions established in the tracery can be seen in Figure 3. So, for example, green diagonals rising from the outer bottom corners of the panel intersect at Christ's chest, passing through the eyes of John the Evangelist, and aligning with Peter's key and Bartholomew's book, at left and right respectively. These attributes, and most of the hands visible in the painting, lie at the level where these green diagonals intersect the orange diagonals of the original double square. The convergence point in Christ's chest also aligns with the fluttering hems of the two angels just beneath his arms, as the blue horizontal there indicates. Taking the distance between the orange and green diagonals as the radius of two blue circles, one begins to find other crucial elements of the composition. The tops of the blue circles, for example, align with the eyes of all of the male saints. Further below, the intersection of the green diagonals with the large yellow arcs defines the horizontal line on which the saints stand. This intersection point is actually picked out at left by the tip of James's sword. Meanwhile, verticals framing the blue circles align with the vertices of triangular gatherings of drapery in the robes of Andrew, Peter, Bartholomew, and Thomas; in the latter case, at right, the drapery gathers

Figure 7 Stefan Lochner, Dombild, with geometrical overlay, stage 1.

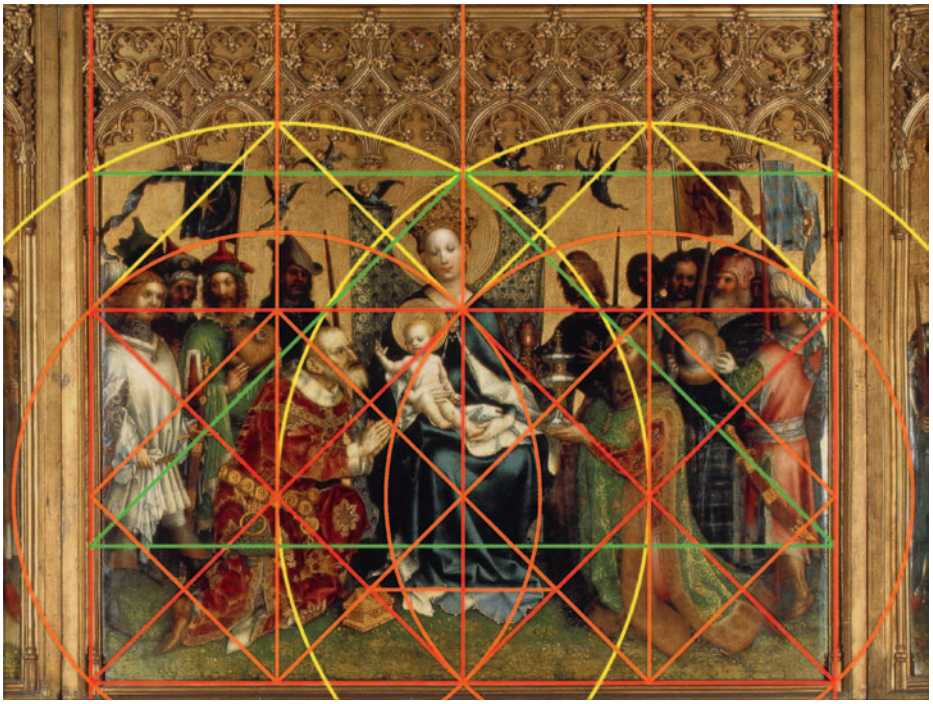
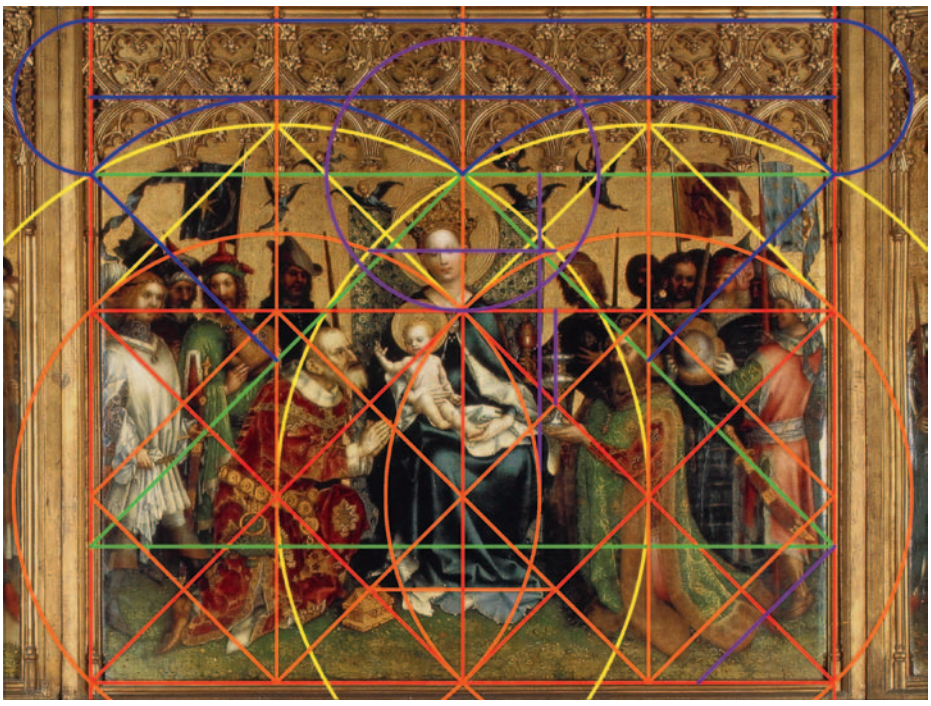


Figure 8 Stefan Lochner, Dombild, with geometrical overlay, stage 2.

to the axis as it falls, instead of spreading out from it, but the overall principle remains the same. Meanwhile, the blue verticals most closely flanking the cross align with the necks of the Virgin, at left, and Saint Paul, at right. A violet circle framed by those verticals and centered on Christ's chest descends to his crotch, passing also through the head of John the Evangelist, and seeming to define its angle of inclination. A larger violet circle struck down to the green equator is just tangent to the heads of Peter and Bartholomew. Its diameter also corresponds to the breadth of the cross.

Returning to the traceried frame from which we started, we can see that the position and orientation of the angels relate closely to the geometry of the gables. The four outer angels look almost as if they were being run over by the violet circles describing the curvature of the arches, while the heads of the two inner angels coincide with the bottom points of the

Figure 9 Stefan Lochner, Dombild, with geometrical overlay, stage 3.



circles flanking the central arch. The lower edge of the cross's horizontal beam, finally, can be found by striking a diagonal down within these violet circles, as shown in the leftmost and rightmost examples. This operation, like the construction of the large yellow arcs at the bottom of the image, corresponds with the principle of square rotation or quadrature often used in Gothic architectural design.

In sum, then, it becomes clear that the overall organization of the crucifixion panel connects closely to the geometrical logic of its traceried frame, and with the principles of contemporary architectural design more generally. Even the position of seemingly minor elements like angels and saints' attributes relate closely to the logic of the panel's geometrical armature. These findings thus raise intriguing questions about the working relationships between painters like the Veronica Master, the carvers responsible for the shrinework that framed their paintings, and the architectural culture of the day.

To see how widespread such relationships may have been in fifteenth-century Cologne, it is helpful to look at slightly later works, such as those associated with the circle of Stefan Lochner. The panel showing Saint Barbara flanked by Saints Mark and Luke, for example, has an elegantly traceried frame with three gabled arches above and six openwork quatrifoils below.[11] As one can see in Figure 4, there is a clear divide between the intricately worked surface behind the saints and the flat gold surface between the gablets. And, as the red lines in figure show, the space above this division is a perfect double square. The orange lines in the figure divide the baselines of each main square into eighths. The small module thus defined sets the height from the red baseline to the base of the pinnacle shafts above their consoles. The outer framing pinnacles are established on bases that correspond to these small square cells, as the yellow square outlines show. Their thickness actually corresponds to the shaded yellow squares inscribed within the outer cells by quadrature, as the small yellow circles between the two squares indicate. This subtle distinction matters, because the overall composition of the panel is governed in part by the margins of the shafts themselves, which are shown by the yellow verticals in Figure 4.

As the green lines in the upper part of Figure 5 demonstrate, the geometry of the tracery framing the saints is quite precise. The axes of the outer pinnacles rise from the centers of the yellow cells just described. The space between these axes is divided into equal thirds by the axes of the two intermediate pinnacles; this is proven by the way the green diagonals spanning the whole space intersect with the green diagonals crossing each half of this space. The axes of the intermediate pinnacles cross the red diagonals of the original generating squares right at the base of the triforium-like arcade at the top of the panel, as the upper green horizontal indicates.

Moving into the lower portion of Figure 5, one begins to see how the widths established in the tracery zone go on to influence the whole composition. The yellow verticals carried down from the pinnacle margins perfectly frame the row of six quatrifoils at the bottom of the image. The blue verticals carried down from the outer edges of the tracery zone, meanwhile, frame a large square whose bottom margin acts as a baseline on which the three saints stand. Within this square, a rotated square can

be inscribed, whose lower left facet aligns with the hem of Barbara's cloak. The blue diagonals of the main square, meanwhile, pass through the heads of the two male saints, intersecting at a point on Barbara's cloak from which a V-shaped gathering of folds falls. Even more strikingly, the upper right diagonal aligns with the upper edge of the small painted panel held by Luke, and with the edge of the baseplate beneath Barbara's miniature tower. These relationships suggest that diagonals like these might actually have been drawn on the panel as its proportions were being established. This conclusion finds further support in the way the bottom of the panel was articulated.

The lower portion of Figure 6 shows that the depth of the panel below the saints' feet can be set by swinging an arc concentric with the main blue square, and with radii given by the violet lines that connect its center to the points on its bottom margin that are cut by the yellow verticals falling from the pinnacles above.[12] From the corners of that new violet baseline, sloping violet lines can then be struck back to the center of the square. These lines intersect the blue diagonals framing Barbara's hem at the level where the brown earth on which the saints stand gives way to the golden surface behind them. The intersection between the left one of these lines and the violet arc is marked by a rock near Mark's feet, while the tip of Luke's pointed shoe lies on the line at right. It should be noted, finally, that some of the most important points in the painting can be located by inscribing circles and squares within each other. So, for example, the eyes of Mark and Barbara lie on the level where a circle inscribed within the large framing square cuts its blue diagonals. The outer shoulders of the two male saints at least roughly follow the contours of this circle, and the main fold lines in their garments align with the violet verticals descending from the intersection points in their heads.

It seems, therefore, that the members of Lochner's workshop were conversant with the principles of square rotation and quadrature seen in contemporary Gothic architecture, and that the painters must have worked very closely with the designers of the panel's traceried frame, whose geometrical logic relates intimately to that of the panel itself. Further evidence for these conclusions comes from consideration of two other paintings even more famous than the two discussed so far: Lochner's Madonna in the Rose Bower, and his altarpiece of the Virgin with the patron saints of Cologne, also known as the Dombild.[13] Bouleau convincingly demonstrated that the geometrical structure of the Madonna in the Rose Bower was based on the figure of two interlocking pentragrams.[14] Such geometries were unusual in the early phases of Gothic architecture, but they were becoming fairly common by the middle of the fifteenth century.[15] The geometry of the Dombild, meanwhile, appears to have been based largely on square rotation.

Lochner's great civic altarpiece, which originally stood in a chapel adjacent to Cologne's City Hall, shows in its central panel the Virgin flanked by the Magi and their entourage (Figure 7).[16] Here, once again, we are fortunate to have the original architectural elements of the frame, which provide helpful data for geometrical analysis. As in the previous cases illustrated in this essay, it makes sense to begin by noting the width between the axes of the pinnacles flanking the panel. This time, however, we can begin the analysis from the ground line so clearly defined by the frame. Figure 7 shows that red diagonals rising from its outer corners converge at the neckline of the Virgin's dress, passing along the way in suggestive parallels to the arms and robes of the two Magi who flank her. A red double square constructed around this figure serves as a podium of sorts for the heads of all the figures in the background. These heads are also grouped at least roughly under the arcs of orange circles circumscribed around the two squares, while the overlap of the two circles defines the almond-shaped space of the Virgin's body. This may all seem rather approximate, but many more precisely defined relationships show that the geometries of the Dombild were as carefully worked out as those of the two panels considered previously. Constructing rotated squares within the two main squares, for instance, we find that the red and orange diagonals intersect exactly on the gifts of the two senior magi. And, the orange vertical that subdivides the left-hand half of the panel rises directly through the body of the leftmost angel. As in the crucifixion panel by the Veronica Master, not only the position but also the orientation of the angels reflects the shape of the geometrical armature.[17]

When yellow diagonals are extended from the corners of the two main squares as in Figure 8, we find that the two inner ones align closely with the wings of three angels flying near the Virgin's head, while the final diagonal aligns with the fluttering pattern of the two flags at right. The intersections of the yellow diagonals can be taken as the corner points of large yellow circles concentric with the two original generating squares. The overlap of these circles defines a large mandorla form, or *vesica piscis*, around the body of the Virgin. The upper right arc of this almond-shaped figure defines the location and dramatic head tilt of the youngest magus. The top point of the mandorla also defines another level, shown as a green horizontal in Figure 8, on which the corbels of the traceried arches stand. Green diagonals descending from its midpoint to the outer framing verticals locate a point at right indicated by the tip of the pink-clad retainer's sword; the same level corresponds to the concave corner in his master's green vestments, and to the crumpled pattern of the Virgin's hem on the ground, as the lower green horizontal shows.

The traceried arches atop the Dombild emerge from the same basic geometrical framework, as Figure 9 reveals. The

green diagonals just described intersect with the orange verticals that divide the panel into four vertical slices, establishing points that can be taken as the centers of blue arcs that reach from the top point of the mandorla around the Virgin up to the blue horizontal in the middle of the gabled tracery zone. Doubling the interval between this blue line and the green horizontal beneath it, therefore, one finds the top margin of the whole composition.

Further relationships can readily be found within this overall geometrical framework, as the violet lines in Figure 9 begin to indicate. The eyes of the Virgin, for example, fall on the level where the orange and yellow circles intersect. The right margin of the cloth of honor behind her aligns with the right edge of the small orange mandorla that frames her lap. And the circle centered at the top of the larger yellow mandorla and resting on the original double square has curving sides that pass through the angels flying near the Virgin's head, calling attention once again to their role as geometrical markers.

Together, the three analyses presented in this essay suggest several provisional conclusions about the geometrical structure of Cologne's fifteenth-century panel paintings. First, the painters in the circles of Stefan Lochner and the Veronica Master seem to have used very similar geometrically-based planning strategies, attesting to a high degree of continuity in local panel production. Second, and more surprisingly, their methods seem to have involved a close dialog between the arrangement of the figures on the panels and the format of their traceried frames. In all three of the paintings just considered, the overall dimensions of the panels relate directly to the spans between the axes of the pinnacles in the frames. In each case, moreover, these axes seem to have had real conceptual priority, defining spaces within which techniques such as square rotation could be used to dynamically unfold more complex formal relationships in the plane.

On one level, these conclusions can stand on their own, since they are based directly on observation of the panels' formal arrangement, but on another level their worth depends on their integration within a larger field of scholarly conversation. It is noteworthy, for example, that the plane geometry used in the layout of the Cologne panels has much in common with the methods used by Gothic architects in the design of full-scale buildings. This approach obviously differs from the approach stereotypically associated with Italian Renaissance painting, in which perspectival geometry played an important role. It may well be, though, that some of these basic techniques of planar arrangement enjoyed currency on both sides of the Alps. That is certainly the view expressed half a century ago by Charles Bouleau, whose extremely broad-ranging study of geometry in painting includes many interesting and potentially valuable observations.[18] His analyses, and those presented here, should be critically cross-examined by specialists who can rigorously assess their relationship to documented practices of painting production. This approach should in due course help to explain how painters, panel-makers, and designers of architectural frames worked together to produce ensembles with the geometrical coherence that we observe.

Notes

1 GHYKA 1946.

2 The special ratio known as the Golden Section, or Φ, harmonically relates the whole to the part in accord with the equation whole/part=part/(whole-part). Writing this as $\Phi/1=1/(\Phi-1)$, or $\Phi^2-\Phi-1=0$, one can solve for $\Phi=(1+\sqrt{5})/2=1.618\ldots$ using the quadratic formula. The length Φ can also be constructed geometrically with a compass by taking a square, and striking an arc from the midpoint of one side through the opposite corner until it aligns with the original side. This is a technique that medieval draftsmen and painters could readily have used.

3 BOULEAU 1963.

4 HECHT's "Maß und Zahl in der gotischen Baukunst" first appeared as three successive issues of "Abhandlungen der Braunschweigischen Wissenschaftlichen Gesellschaft": 21 (1969), 22 (1970), and 23 (1970). The complete study has been republished as a single volume by Georg Olms Verlag (Hildesheim, 1979).

5 BORK 2011.

6 For a more in-depth presentation of the geometrical argument sketched here, see BORK 2011, pp. 110-121. The most detailed study of Plan F is STEINMANN 2003.

7 In the drawing known as Rahn Riss A, for example, the closely spaced arcs depicting the inner moldings of a flying buttress were surely drawn with a compass, but there is no compass prick at their geometrical center. This suggests that its creator may have protected the parchment of his drawing by placing the compass point on a temporary shielding sheet. The drawing is preserved in the Archives de l'État in Fribourg, Switzerland.

8 I am grateful to Roland Krischel of the Wallraf-Richartz Museum, and to Norbert Nussbaum of the University of Cologne, for suggesting that I explore this material.

9 In the course of his wide-ranging study, Bouleau provides perceptive analyses of northern fifteenth-century paintings by artists including the Limbourgs, Rogier van der Weyden, Jean Fouquet, the Master of Moulins, and Enguerrand Quarton, as well as Stefan Lochner; see BOULEAU 1963, pp. 40-79. His findings now strike me as very compelling, but they differ from mine in at least

two respects. First, while his analyses often consider the proportions of the rectangular frames used by these artists, he did not engage with traceried architectural framing elements like those that I took as my point of departure in studying the Cologne panels. Second, he persuasively demonstrates the widespread use of proportions based on Golden Section Φ and the associated symmetry of the pentagon, which I had not discerned in the Cologne panels I examined.

10 The painting, which holds inventory number WRM 14, measures 176 x 245 cm. See STEFAN LOCHNER 1993, p. 296.

11 The painting, which holds inventory number WRM 68, measures 100.5 x 58 cm. See STEFAN LOCHNER 1993, p. 320.

12 This compositional principle of extending the rectangular frame recalls the one cited by Bouleau in the case of Rogier van der Weyden's Descent from the Cross. See BOULEAU 1963, p. 67. It is noteworthy that the violet horizontal tracks across the top of the quatrifoils just as the yellow verticals track across the sides of the lateral quatrifoils.

13 The Madonna in the Rose Bower holds inventory number WRM 67. See STEFAN LOCHNER 1993, p. 330.

14 BOULEAU 1963, p. 69.

15 Pentagonal symmetries are already seen around 1200 in the planning of large-scale structures like the choir of Reims Cathedral, but the pentagon became a common motif in small-scale structures such as window tracery only about a century later. Pentagons are notoriously more difficult to construct than squares, octagons, or equilateral triangles, and it is interesting and significant that Mathes Roriczer was still using an imperfect approximate construction of the pentagon in his "Geometria Deutsch" in the 1480s, and Dürer republished it alongside the correct construction in his "Unterweysung" of 1525. See CROWE 1992.

16 The painting, which now stands in the Marienkapelle in the south choir aisle of Cologne Cathedral, measures 260 x 285 cm across its central panel. See STEFAN LOCHNER 1993, p. 324.

17 The same use of wings as geometrical signals is seen even more clearly in the Coronation of the Virgin painted by Enguerrand Quarton, as demonstrated in BOULEAU 1963, p. 71. In that instance, the wings of the dove of the Holy Spirit fit precisely into the space between two rotated pentagons defining the central composition, while the horizontally extended wing of an angel at right aligns with the top edge of these interlocking figures.

18 For his discussion of the intersection between planar and perspectival composition strategies, see BOULEAU 1963, pp. 81-145, especially p. 97 on Piero della Francesca.

Literature

BOULEAU 1963
Charles Bouleau, *The Painter's Secret Geometry: A Study of Composition in Art*, New York 1963

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CROWE 1992
Donald Crowe, "Albrecht Dürer and the Regular Pentagon," in: István Hargittai (ed.), *Five-fold Symmetry,* London 1992

GHYKA 1946
Matila Ghyka, *The Geometry of Art and Life*, New York 1946

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Konrad Hecht, *Maß und Zahl in der gotischen Baukunst*, Hildesheim 1979

STEFAN LOCHNER 1993
Stefan Lochner Meister zu Köln: Herkunft-Werke-Wirkung, exhibition catalog, Wallraf-Richartz Museum, Cologne, 3 December 1993-27 February 1994, Cologne 1993

STEINMANN 2003
Marc Steinmann, *Die Westfassade des Kölner Domes: Der mittelalterliche Fassadenplan F*, Cologne, 2003

Photo Credits

Fig. 1
Source image from: Marc Steinmann, *Die Westfassade des Kölner Domes: Der mittelalterliche Fassadenplan F*, Cologne 2003, Fig. 1. Geometrical scheme of the façade by the author.
Fig. 2-6
Source images kindly provided by Norbert Nussbaum. They are rectified images of WRM 14 and WRM 68 made by Ulrich Jacobs.
Fig. 2-9
Computer graphics by the author.
Fig. 7-9
Source image from: Julien Chapuis, *Stefan Lochner: Image Making in Fifteenth-Century Cologne,* Turnhout 2004, Fig. 20.